Isospin Blockade in Transport through Vertical Double Quantum Dots

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Abstract

We study the spectrum and the transport properties of two identical, vertically coupled quantum dots in a perpendicular magnetic field. We find correlation-induced energy crossings in a magnetic field sweep between states differing only in the vertical degree of freedom. Considering the influence of a slight asymmetry between the dots caused by the applied source-drain voltage in vertical transport experiments these crossings convert to anticrossings accompanied by the build-up of charge polarization which is tunable by the perpendicular magnetic field. The polarization strongly affects the vertical transport through the double quantum dot and is manifest in an isospin blockade and the appearance of negative differential conductances in the magnetic field range where the charge localization occurs.

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Double quantum dots have recently attracted much interest as possible implementations of qubits and as model systems to study the few-particle physics of molecular binding [1,2,3]. In vertically coupled double quantum dots (DQD) the molecular binding for a single electron is determined by the vertical degree of freedom, while the lateral degree of freedom reflects the physics of the single dots or quasiatoms building up the molecule. In this paper we describe level crossings between few-particle states differing only in parity along the vertical direction. These crossings are based on Coulomb correlations between the electrons and may lead to strong charge polarization, which is tunable by an external perpendicular magnetic field[4]. Thus, these crossings mark changes of the molecular binding which originate from many-body effects.

Besides its importance for quantum dot physics, the effect illuminates the analogy between real spin and isospin, the latter of which describes the vertical degree of freedom[1]. This analogy manifests itself in an isospin blockade of transport through the double dot system[5,6,7]. Furthermore, the localization provides an effective two-level system which may be useful as a field-tunable charge qubit.

We model the DQD in vertical direction by two parallel layers separated by a distance d. In lateral direction the electrons are confined by a rotationally-symmetric parabolic potential of strength $\hbar\omega_0$. Therefore, the vertical degree of freedom is reduced to an additional spin-like degree of freedom, the isospin[1]. In analogy to the real spin one can define a spin algebra for the isospin, where the z-component I_z specifies the vertical degree of freedom. An electron with $I_z = +1/2$ ($I_z = -1/2$) is situated in the upper (lower) dot. We

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perform an exact diagonalization of the few-particle Hamiltonian including tunneling between the dots as well as the Zeeman term and the full Coulomb interaction. Thus, correlations between electrons are fully taken into account. The strength of tunneling t is specified by the energy splitting between the symmetric and antisymmetric single particle states in z-direction, $t = -\Delta_{SAS}/2$. Due to the symmetries of the system the z-component of the angular momentum, M, the magnitude of the total spin, S, and the z-component of the spin, S_z are conserved. If both dots are identical and no asymmetry between the layers is present, an additional symmetry leads to the conservation of the isospin parity, $\hat{P} = 2^{N_e} \cdot \hat{I}_x^{(1)} \otimes \ldots \otimes \hat{I}_x^{(N_e)}$ specified by the quantum number $P = \pm 1$. The isospin parity flips all isospins, e.g. it moves all electron orbitals from the upper dot to the lower dot and vice versa. Increasing the vertical magnetic field effectively leads to a stronger lateral confinement of the electrons. Therefore, the Coulomb energy increases with the magnetic field which causes ground state (GS) crossings between states differing in angular momentum and/or spin [1].

In this paper we discuss energy crossings in a magnetic field sweep between states differing only in parity, e.g. in the vertical degree of freedom [4]. These crossings originate from the different scaling with magnetic field of the *intradot* Coulomb interaction that accounts for the interaction between electrons on the same dot and *interdot* Coulomb interaction acting between electrons on different dots. The intradot interaction increases faster with increasing magnetic field than the interdot interaction, that is limited to $1/d$ [1]. Hence, the Coulomb correlations in the eigenstates of the DQD are magnetic field dependent.

In the following we study parity crossings in a DQD containing three electrons. For small tunneling $t \ll \hbar\omega_0$ the energy splitting between the parity eigenstates $|P = \pm 1\rangle$ within the same set of quantum numbers (M, S, S_z) is due to their different tunneling energies, as the occupation probabilities of symmetric and antisymmetric orbitals depends on parity. However due to the magnetic-field-dependent Coulomb correlations the favored parity may change with magnetic field. In the example discussed here $P = -1$ is preferred for magnetic fields $B < 7.75$ T whereas $P = 1$ is preferred for strong magnetic fields, $B > 7.75$ T.

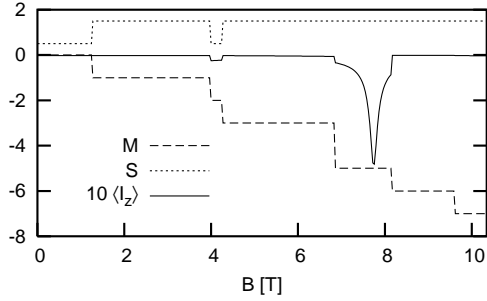Parity crossings between states with the same quantum numbers M, S and S_z can have drastic effects.

Fig. 1. Angular momentum M, total spin S and expectation value of the z-component of the isospin $\langle I_z \rangle$ for the three-electron GS. The peak in $\langle I_z \rangle$ illustrates the charge localization that corresponds to a parity anticrossing (see text). Parameters [11]: Tunneling strength: $t = -0.059$ meV, confinement: $\hbar\omega_0 = 2.96$ meV, layer separation: $d = 19.6$ nm, asymmetry: $V_z = 5.9 * 10^{-4}$ meV.

Fig. 1 illustrates the characteristics of the three-electron ground state (GS) in a slightly asymmetric DQD as function of magnetic field. An asymmetry between the dots is modeled by a small voltage drop between them. We account for it by adding the term $\hat{\mathcal{V}}_z = V_z \cdot I_z$ to the Hamiltonian, where $I_z = \sum_{i=1}^{3} I_z^{(i)}$ is the z-component of the total isospin and V_z is assumed to be much smaller than all other parameters $V_z \ll t, \hbar\omega_0$. At around $B = 7.75$ T there is a sharp peak in the average isospin z-component, corresponding to a strong charge polarization. Right at the peak, two electrons occupy the lower dot and only one electron occupies the upper one. The charge localization is a consequence of a parity crossing in the symmetric DQD, that turns into an anticrossing due to the slight asymmetry between the dots. The width of the peak and the energy splitting at the anticrossing are proportional to the strength of asymmetry[4]. For the calculation presented in Fig. 1 we chose the parameters such, that the anticrossing occurs in the GS of a DQD containing three electrons. But the effect of charge localization is neither bound to this particular choice of parameters nor to three electron states. We found similar effects also for 5 electrons and/or in other subsets of quantum numbers M, S and S_z.

In the following we show that the charge localization drastically affects the transport through the DQD. We assume a vertical transport setup where the current and the differential conductance through the DQD are measured as response to a source-drain voltage V_{sd} and a gate voltage V_G [3,8,10]. The source-drain voltage

determines the width of the transport window where electrons from the source reservoir can tunnel through the DQD to the drain reservoir, whereas the gate voltage changes the energies of the DQD proportionally to $V_G N$ where N denotes the number of electrons in the DQD. The energy needed for a transition between an N-electron state i with energy $E(N, i)$ and an $N + 1$-electron state j with energy $E(N + 1, j)$ is therefore given by $\mu(N + 1, j; N, i) = E(N + 1, j) - E(N, i) - e\alpha V_G$, where α denotes a constant proportionality factor. Hence the gate voltage together with the eigenenergies of the DQD determine which transitions lie within the transport window. On the other side the value of the transition rates depends on the form of the eigenstates. In special cases a transition is forbidden since it violates conservation laws. A well-known example is the spin-blockade that occurs if the spins of the two states differ by more than $\Delta S = \pm\frac{1}{2}$ [5]. For general cases the effect of the structure of the wavefunctions enters the transition rates through the spectral weights [9] that determine the probability for an electron entering a N-electron state of the DQD to cause a transition to a $N + 1$-electron state of the DQD.

In our calculations we assume the coupling of the DQD to the external reservoirs to be weak in comparison with the average energy spacing in the DQD. Consequently the coupling to the external reservoirs is treated to lowest order perturbation theory and the transport through the DQD is described by sequential-tunneling processes in and out of many-particle eigenstates of the isolated DQD [9]. Therefore, tunneling causes transitions between those eigenstates of the DQD which differ by one in the number of electrons. We assume that an electron in the upper (lower) reservoir can only tunnel into the upper (lower) dot, which leads to different spectral weights for the tunneling-in and tunneling-out process [9]. Assuming the source reservoir to be the upper reservoir, the spectral weights for a transition between the i-th two-electron state and the j-th three-electron state caused by a tunneling-in (-out) process is given by $\sum_l |\langle N_e = 2, i| d_{l\pm} |N_e = 3, j\rangle|^2$. l runs over all possible single particle states and d_{l+} (d_{l-}) denotes an annihilator of an electron in orbital l in the upper (lower) dot. We include the asymmetry between the dots caused by the source-drain voltage applied across the DQD by setting the asymmetry $V_z = \frac{1}{20}eV_{sd}$. With these assumptions we calculate the stationary

current and the differential conductance through the DQD in a vertical transport experiment by solving the rate equations containing the transition rates and the probabilities of occupying the many particle eigenstate of the DQD [9].

Figure 2 shows a charging diagram where the differential conductance $G = \partial I/\partial V_{sd}$ is plotted for each pair of transport voltage and gate voltage. The conductance shows significant values only for source-drain voltages $V_{sd} > 0.35$ mV. This is surprising since one could expect that the maxima of the conductance form Coulomb diamonds starting from $V_{sd} = 0$ mV and $V_G = 16.1$ mV, since at these voltages the Fermi energies of the external reservoirs align with the energy needed for a GS-GS transition between two and three electrons. The results shown in Fig. 2 resemble the well-known spin blockade[5]. In fact our results can be explained by an isospin blockade due to the charge polarization of the three electron GS. An electron tunneling from the source reservoir (upper reservoir) to the upper quantum dot can hardly cause a transition from the unpolarized two-electron GS to the three-electron GS that has two electron charges in the lower dot. In other words, the two electron state is approximately an eigenstate to $I_z = 0$, which by an electron entering the upper dot turns approximately to an eigenstate to $I_z = \frac{1}{2}$. This state has no overlap with the three electron GS, that is approximately an eigenstate to $I_z = -\frac{1}{2}$. This explains why the spectral weight and therefore the transition rate for the tunneling-in process nearly vanish and illuminates the relation to the spin blockade, which is based on the orthogonality of eigenstates with different spin. Rising the source-drain voltage other transitions enter the transport window that enable transport. The strong suppression of the current discussed here demonstrates again the influence of the structure of the eigenfunctions on the transport characteristics [9].

Another prominent effect of charge localization on the transport properties is the appearance of broad regions of negative differential conductances in the charging diagrams for magnetic field strengths close to the anticrossings. Figure 3 shows the current (upper part) and the differential conductance (lower part) for $B = 7.24$ T. For small transport voltages, the three-electron GS is only weakly polarized and the GS-GS transition between two and three electrons is allowed. Therefore, Fig. 3 shows current already for small transport volt-
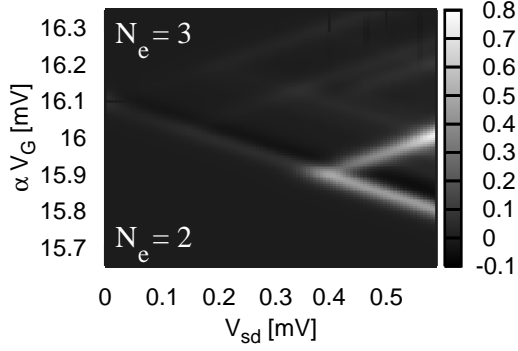
Fig. 2. Section of charging diagram showing the differential conductance $G = \partial I / \partial U_{sd}$ for vertical transport through DQD. $B = 7.74$ T (i.e. three-electron GS is strongly polarized). The conductance is strongly suppressed at low transport voltages, due to an isospin blockade. Units of G: $e\Gamma/mV$, where $\Gamma = DOS \, |T_R|^2 \frac{2\pi}{\hbar}$ determines the coupling to the external reservoirs, DOS is the density of states in the reservoirs, T_R: tunneling matrix elements to the reservoirs. Parameters like in Fig. 1 and temperature $T = 140$ mK.

ages in contrast to Fig. 2. But increasing the source-drain voltage also increases the asymmetry between the dots and therefore the polarization of the three-electron GS, which reduces the spectral weight for the tunneling-in process. Hence the current may decrease with increasing transport voltage giving rise to negative differential conductances.

In conclusion: We observed and explained correlation induced parity crossings in a symmetric DQD in a magnetic field sweep. These crossings convert to a magnetic-field-dependent charge-localization for an odd numbers of electrons inside the DQD if the symmetry between the dots is broken. This localization strongly affects the transport characteristics and leads to an isospin blockade as well as to negative differential conductances. We thank Michael Tews for many illuminating discussions and acknowledge financial support by the Deutsche Forschungsgemeinschaft via SFB 508.
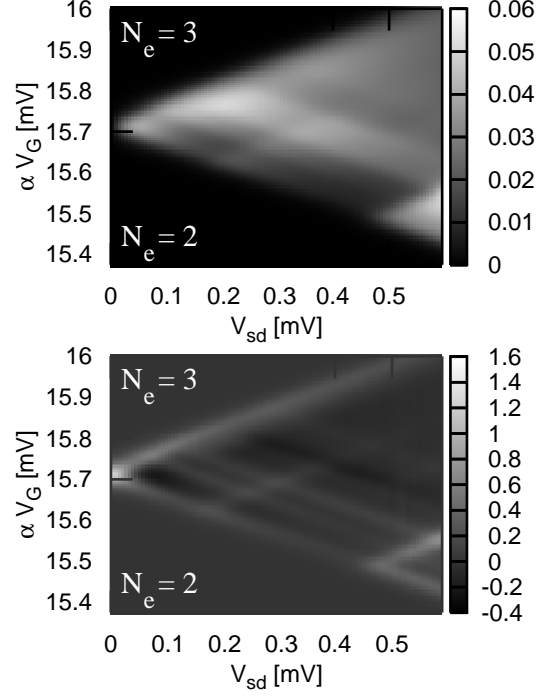

Fig. 3. Charging diagrams for transport through DQD. Asymmetry between the dots increases with increasing transport voltage reducing the current (upper part) and leading to broad regions of negative differential conductance (lower part) indicated by the dark black regions. Current in units of $e\Gamma$, $B = 7.24$ T; Other parameters like in Fig. 2.

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[11] Throughout this paper we have used the GaAs parameters for the effective mass m^* of the electrons and the Landé factor g^*, e.g. $m^* = 0.067 m_e$ and $g^* = -0.44$.